Fundamentals of Charged Particle Optics in High Energy Accelerator Systems

U.S. Particle Accelerator School

Winter 2008

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Class Overview

- Students:
  - 23 in the class
    - ~15 from labs/research centers
    - ~8 from universities; 1, high school
    - Various stages/levels of education:
      - 3 PhD, 4 MS, 5 gs, 9 BS, 2 ug, 1 HS(!)
  - credit (undergraduate) vs. audit
    - 13 - Credit 10 - Audit
    - PLEASE CONFIRM -- initial sheet!

Course Overview

- Charged Particle Optics
  - Beam transport (beam lines)
  - Periodic (circular) systems
  - Mostly, concerned with single particle effects
- Why “high energy” in this course?
- First week: basic, fundamental concepts
- Second week: more details; design issues

Syllabus / Procedures

- lectures, labs, homework, exam
  - lectures in mornings
  - lecture/discussion + lab in afternoon
  - study sessions in evenings
- physics vs. technology
- lots to cover in SHORT time !!

Homework/Labs

- Many problems on the handout; not all to be assigned
- Will choose from the list each day, ~4 problems
  - problems AND labs due 9:00 a.m. next morning
  - will go over HW in afternoon sessions
- Enthusiasts can use 'extra' HW as practice; can go over during future discussion periods...
- Hopefully Labs can be done in 2-hr slot; room also will be available at other times...

Syllabus

Will certainly evolve...

Week 1

Day	9:00 - 10:15	10:30 - 11:45	1:30 - 2:45	3:00 - 5:00
Mon	Introduction & Course Prerequisites	Single Particle Trajectories and Weak Focusing	Linear Guide Fields and Focusing Fields	Lab: Weak Focusing, Optical Elements
Tues	Matrix Formalism and Strong Focusing	Analytical Methods – Courant Snyder Parameters	Courant Snyder Parameters – II	Lab: Doublets, Triplets, Ray Tracing
Wed	Phase Space and Emittance	Off-Momentum Considerations	*Homework Review & Discussion*	Lab: Lattice Parameters
Thur	Transverse Linear Errors and Adjustments	Additional Optics Components	*Homework Review & Discussion*	Lab: Steering and Dispersion
Fri	Optical Design – cells and insertions	Optical Design – off-momentum	*Homework Review & Discussion*	

Week 2

Day	9:00 - 10:15	10:30 - 11:45	1:30 - 2:45	3:00 - 5:00
Mon	Synchrotron Radiation	Light Source Lattices	Beam and Optics Diagnostics	Lab: Optical Insertions
Tues	Sensitivity Analyses	Beam Lines *vs.* Circular Accelerators	*Homework Review & Discussion*	Lab: Beam Line Analysis
Wed	Consideration of Nonlinearities	Space Charge Effects	*Homework Review & Discussion*	Lab: Synchrotron Analysis
Thur	Transverse Coupling	Emittance Exchange	**Review Session**	Lab: *Finish Labs*
Fri	**Wrap Up 9:00-9:30 a.m.**	**Final Exam 10:00 a.m.-noon**		

Today...

- Review of Classical Physics, esp. required concepts
  - Newton, Maxwell, special relativity, ...
- “Weak” Focusing synchrotron
- Linear Guiding and Focusing Magnetic Fields
  - Goals:
    - review (get used to problem solving again!)
    - get feel for techniques, range of parameters

Some Philosophy

- Optics design and calculations
- Modularity of Optical Systems
- Design Codes
- Acknowledgments
- Apologies
    - since from FNAL, SSC, BNL -- many examples are from these labs; will attempt to be general...

Homework for Tuesday

- Problem Set 1 -- Nos. 3, 4, 5, 8, and 10

Invention of Particle Accelerators

- Early DC Accelerators -- van de Graaf, Cockcroft-Walton, ...
- AC Required for higher energies
  - Wideroe and recognition of RF
  - Invention of cyclotron by Lawrence, et al.
  - Radar leads to Alvarez linear accelerator
  - The Betatron principle
  - Invention of the Synchrotron -- McMillan, Veksler
  - Invention of Alternating Gradient Focusing

Modern Accelerators

- The HEP era -- SLAC, CESR, Tevatron, LEP, KEKb, PEP II, SSC, LHC, ...
- Also, modern-day Nuclear Physics -- CEBAF, RHIC
- Emergence of other interests -- medicine, defense, industry -- light sources, etc.
- Someone did a better job ...
  - where do those 1 Joule cosmic rays come from?

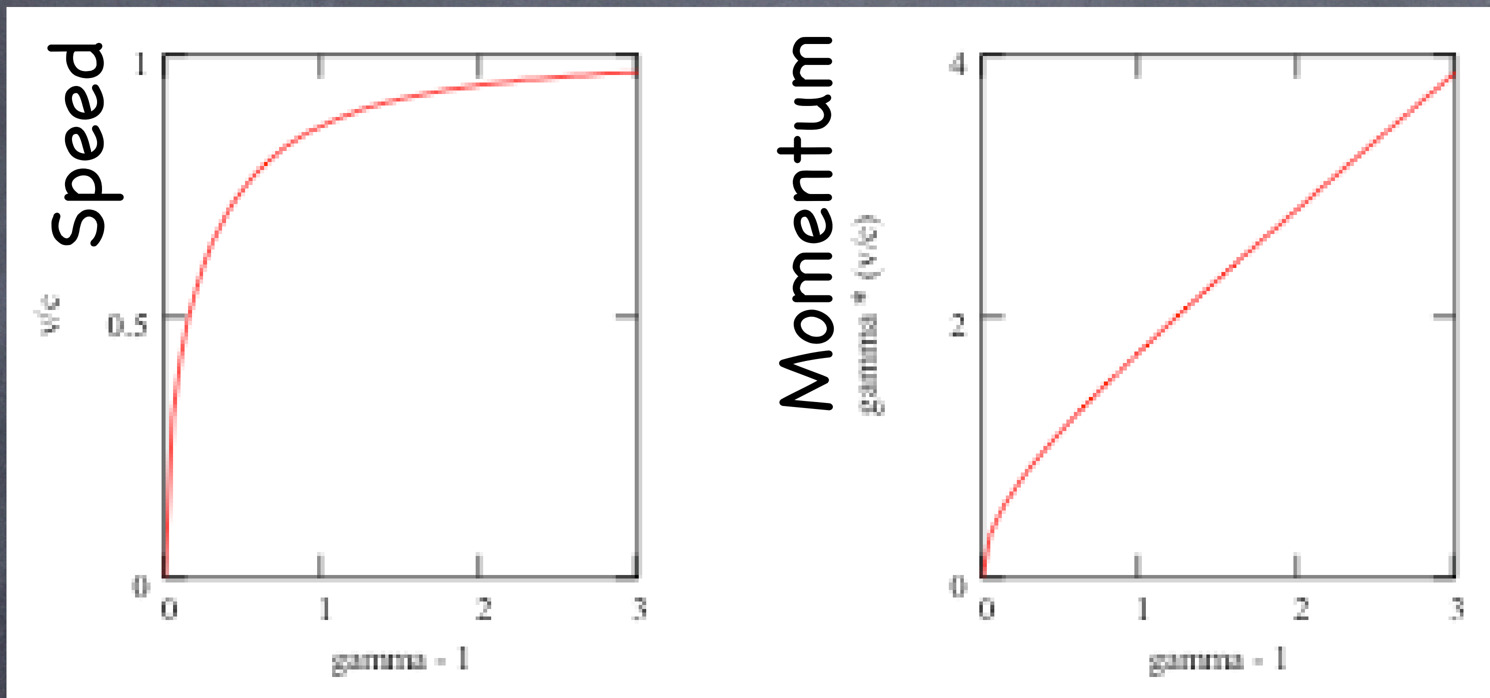
Single Particle Trajectories

- In each case -- single particle motion in magnetic fields
- Newton, Maxwell, Lorentz force, Relativity
- Magnetic rigidity
- The need for transverse focusing
  - “emittance” of a beam
  - space charge force
  - stability of motion
- Electric vs. Magnetic forces on a charged particle

Special Relativity

- Frames of Reference
- The Principle of Relativity
- The Problem of the Velocity of Light
- Simultaneity
- Lengths and Clocks
- The Lorentz Transformation
- $E=mc^2$
- Transformations of E-, B-fields

Speed, Momentum, vs. Energy



Kinetic Energy Kinetic Energy

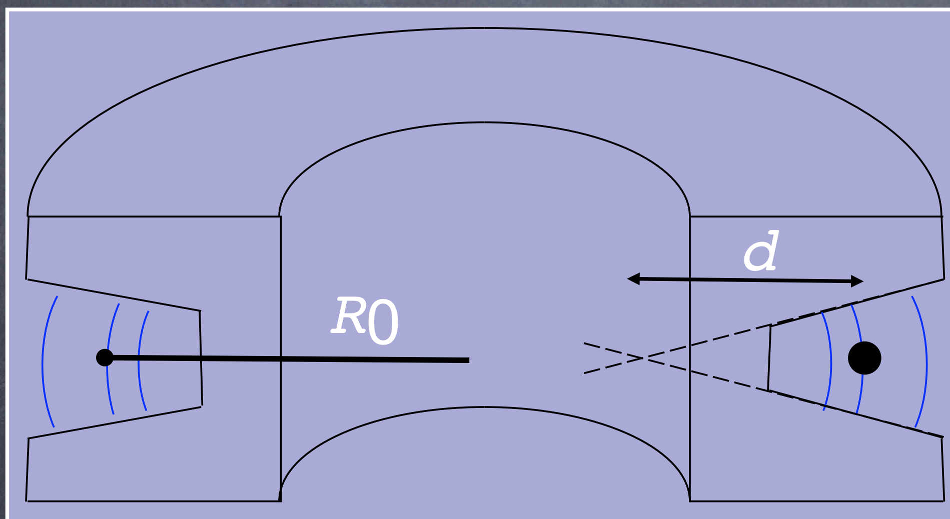
Electron:	0	0.5	1.0	1.5	MeV
Proton:	0	1000	2000	3000	MeV

Single Particle Trajectories

- Newton, Maxwell, Lorentz force $\surd$
- Relativity $\surd$
- Magnetic rigidity
- The need for transverse focusing
  - "emittance" of a beam
  - space charge force
  - stability of motion
- Electric vs. Magnetic forces on a charged particle

Weak Focusing System

(as it has come to be known...)



- Field varies with radius:

$$B = B_0 \left(\frac{R_0}{r}\right)^n$$

$$n \approx \frac{R_0}{d}$$

n is determined by adjusting the opening angle between the poles

$d = \infty,\ n = 0$

$d = R_0,\ n = 1$

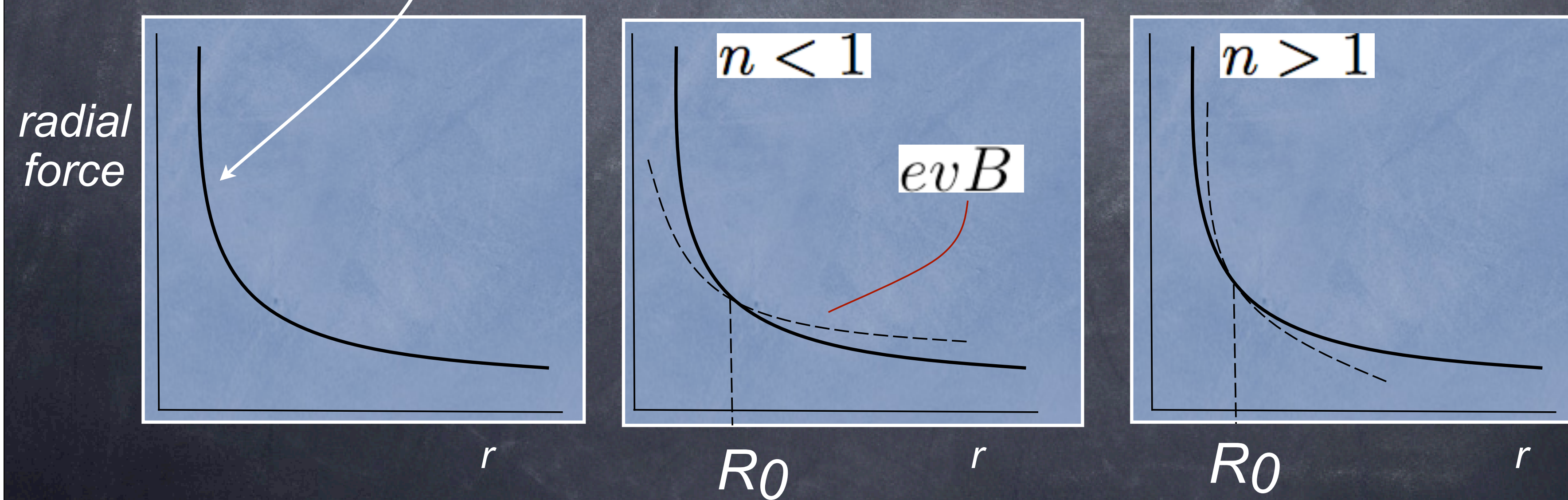
Weak Focusing System

$$B = B_0 \left(\frac{R_0}{r} \right)^n$$

"field index"

Centr. Force: $\frac{mv^2}{r} = evB_0$

$$n \equiv -\frac{R_0}{B_0} \left(\frac{\partial B}{\partial r} \right)_{r=R_0}$$



Weak Focusing System

- Differential Equations (Horizontal and Vertical)
- Betatron Tune
- Stability Condition
- Maximum Oscillation Amplitude

Weak Focusing:

Differential Equations

Radial:

$$
\begin{aligned}
\gamma m(\ddot{r} - r\dot{\theta}^2) &= -evB_y = -evB_0\left(1 - n\cdot\frac{x}{R_0}\right) \\
\gamma m\ddot{r} &= \gamma m\frac{v^2}{r} - evB_0\left(1 - n\cdot\frac{x}{R_0}\right) \\
\ddot{r} &= \frac{v^2}{R_0}\left(1 - \frac{x}{R_0}\right) - \frac{ev^2B_0}{\gamma mv}\left(1 - n\cdot\frac{x}{R_0}\right) \\
\ddot{x} &= \frac{v^2}{R_0}\left(1 - \frac{x}{R_0}\right) - \frac{v^2}{R_0}\left(1 - n\cdot\frac{x}{R_0}\right) \\
\ddot{x} &= -\left(\frac{v}{R_0}\right)^2(1-n)x
\end{aligned}
$$

$$\ddot{x} + \omega_0^2(1-n)x = 0$$

Vertical:

$$
\begin{aligned}
\gamma m\ddot{y} &= evB_x \\
&= evB_0\left(-n\cdot\frac{x}{R_0}\right) \\
\ddot{y} &= \frac{ev^2}{\gamma mv}B_0\left(-n\cdot\frac{x}{R_0}\right) \\
&= -n\left(\frac{v}{R_0}\right)^2 x
\end{aligned}
$$

$$\ddot{y} + \omega_0^2 nx = 0$$

must have
$0 \le n \le 1$
for stability

- Betatron Tune
  - # osc.'s per turn: $\nu_x = \sqrt{1-n}, \quad \nu_y = \sqrt{n}$

Maximum Excursions

- Solution is Simple harmonic Oscillator:

$$\begin{aligned} x &= \text{displacement from} \\ &\quad \text{design trajectory} \\ x' &= dx/ds = \text{slope} \\ &\quad \text{w.r.t. design trajectory} \end{aligned}$$

$$\begin{aligned} \ddot{x} + \omega_0^2 \nu^2 x &= 0 \\ v^2 x'' + \omega_0^2 \nu^2 x &= 0 \\ x'' + \left(\frac{\nu}{R_0}\right)^2 x &= 0 \end{aligned}$$

$$\Longrightarrow$$

$$x(s) = x_0 \cos\left(\frac{\nu}{R_0} s\right) + x_0' \frac{R_0}{\nu} \sin\left(\frac{\nu}{R_0} s\right)$$

- For given angular deflection, Maximum Excursion:

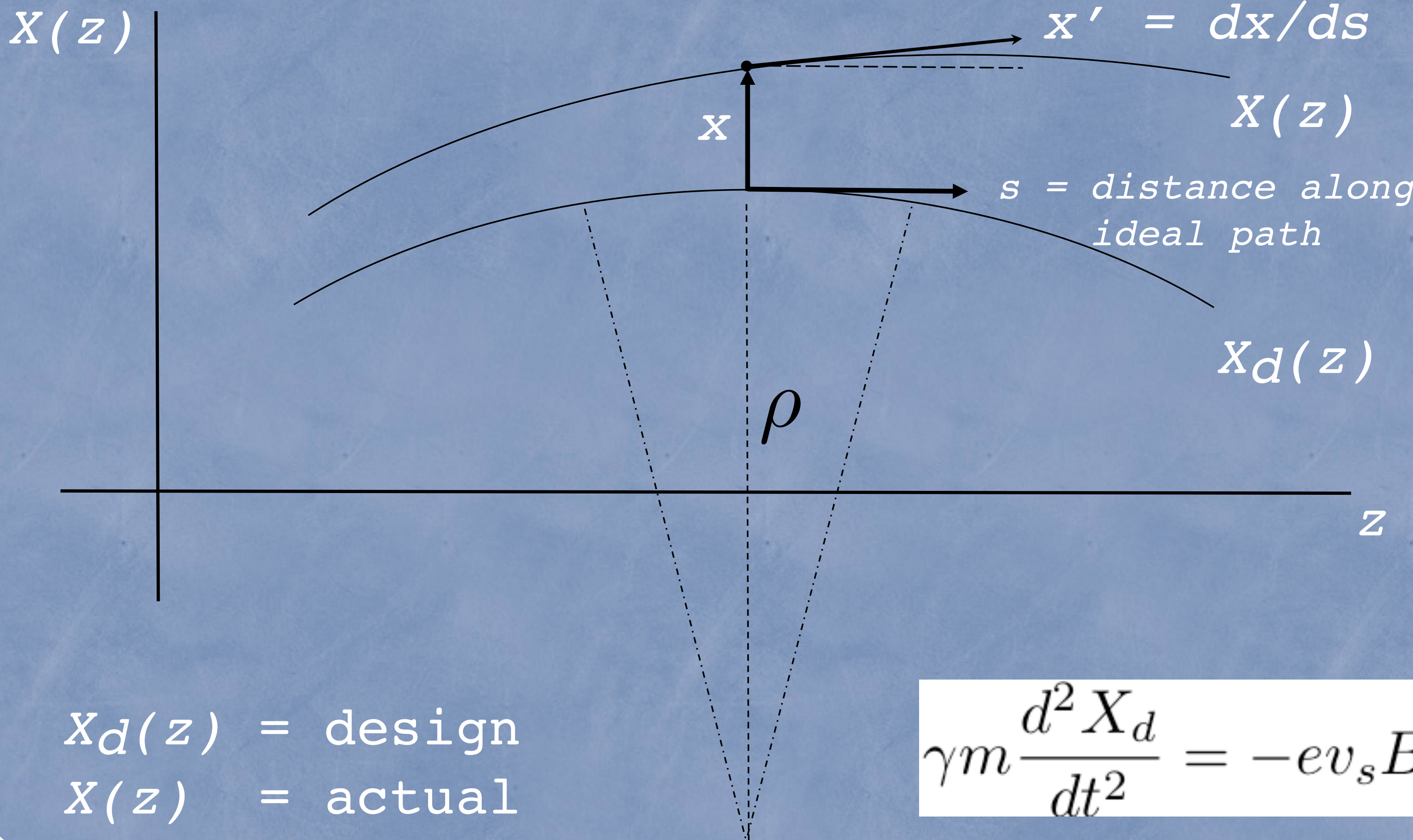
$$\boxed{x_{max} = \frac{R_0}{\nu} x_0'}$$

  - Note: 0 < tune < 1,
  - Thus, due to limited range of *n*, then as *R* (*i.e.*, energy) got large, so did the required apertures

Linear Guiding and Focusing

- Desire particles "near" the design trajectory to remain near the design trajectory -- as we saw in weak focusing system, for small displacements, want a restoring force proportional to displacement
  - Result: Simple Harmonic Motion
- Will explore the use of linear fields
  - B = constant B = constant x displacement

Guide Fields and Linear Focusing Fields



$X_d(z)$ = design
$X(z)$ = actual

$$\gamma m \frac{d^2 X_d}{dt^2} = -e v_s B_0$$

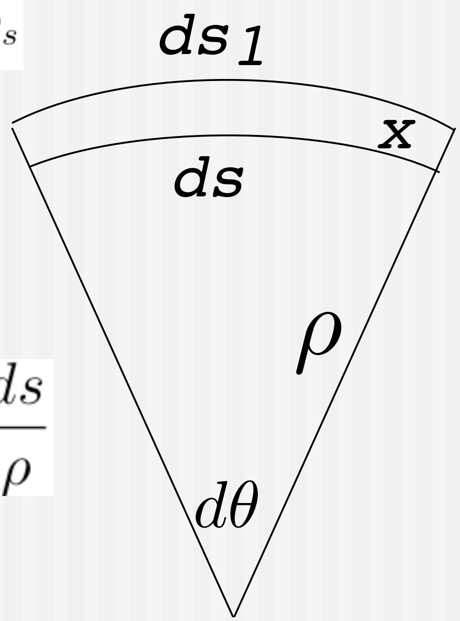
Linear Restoring Forces

- Assume linear guide fields: --

$$B_y = B_0 + B'x$$
$$B_x = B'y$$

- Look at radial motion:

$$\frac{dx}{dt} = \frac{dx}{ds}\frac{ds}{dt} = x' v_s$$



$$\frac{ds_1}{\rho + x} = \frac{ds}{\rho}$$

$$\begin{aligned}
\gamma m \frac{d^2(X_d)}{dt^2} &= -e v_s B_0 \\
\gamma m \frac{d^2(X_d + x)}{dt^2} &= -e v_{s1} B_y(X) \\
\gamma m (X_d'' + x'') v_s^2 &= -e v_{s1} B_y(X) \\
\gamma m v_s x'' &= -e \frac{v_{s1}}{v_s} B_y + e B_0 \\
\gamma m v_s x'' &= -e \left[B_y \left(1 + \frac{x}{\rho} \right) - B_0 \right] \\
x'' &= -\frac{e}{p} \left[(B_y - B_0) + B_y \frac{x}{\rho} \right] \\
&\approx -\frac{1}{B\rho} \left[B'x + B_0 \frac{x}{\rho} \right]
\end{aligned}$$

Hill's Equation

- Then, for vertical motion:

$$
\begin{aligned}
\gamma m \frac{d^2(Y_d)}{dt^2} &= 0 \\
\gamma m \frac{d^2(Y_d + y)}{dt^2} &= e v_{s1} B_x(Y) \\
\gamma m v_s^2 y'' &= e v_{s1} B_x(Y) \\
\gamma m v_s y'' &= e \frac{v_{s1}}{v_s} B_x \\
\gamma m v_s y'' &= e B_x \left(1 + \frac{x}{\rho}\right) \\
y'' &= \frac{e}{p} \left[B_x (1 + \frac{x}{\rho}) \right] \\
&\approx \left(\frac{B'}{B\rho}\right) y
\end{aligned}
$$

  - So we have,

    to lowest order,

$$
\begin{aligned}
x'' + \left(\frac{B'}{B\rho} + \frac{1}{\rho^2}\right) x &= 0 \\
y'' - \left(\frac{B'}{B\rho}\right) y &= 0
\end{aligned}
$$

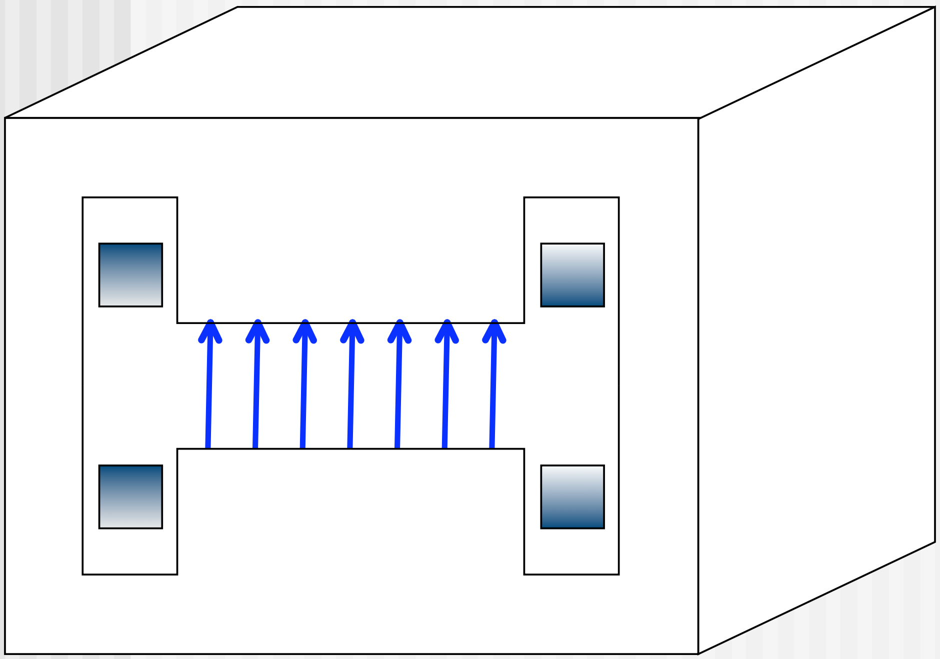
General Form: $\longrightarrow$ $\boxed{x'' + K(s)x = 0}$

Hill's Equation *(SHO, for K=const.)*

Beam Line Components

- Electrostatic deflectors and the need to use magnetic fields
- Iron-dominated Magnetic Elements
  - dipole, quadrupole, n-pole magnets
  - combined function magnet
  - Lambertson magnets, solenoids, kickers, ...
  - Hysteresis in iron magnets
- Solving Poisson's Equation by "relaxation"

Magnetic Elements



$B_y = B_0$

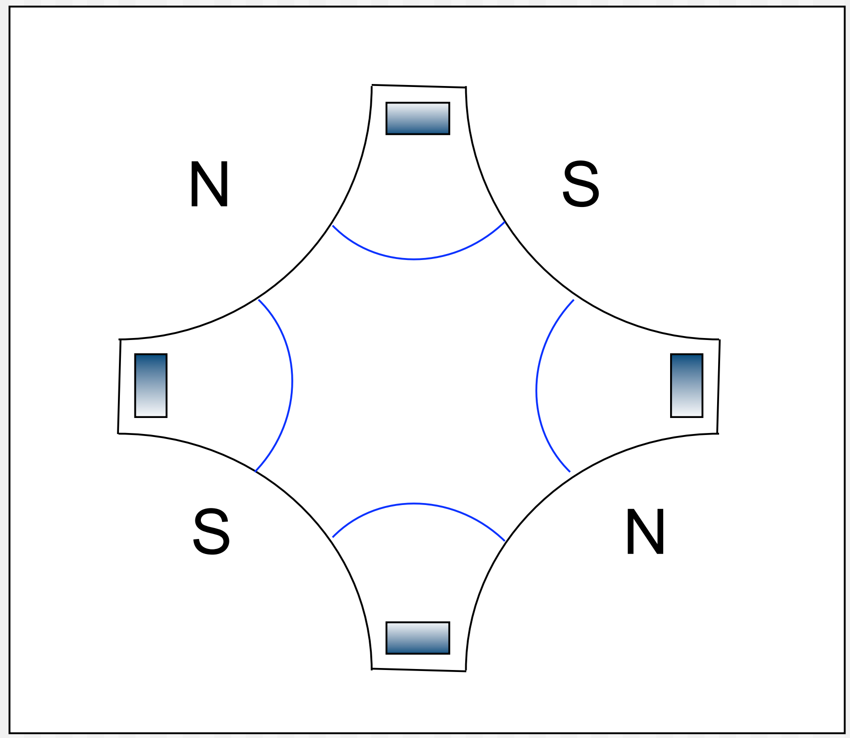
$B_x = 0$

(Dipole Magnet)

$B_y = B'x$

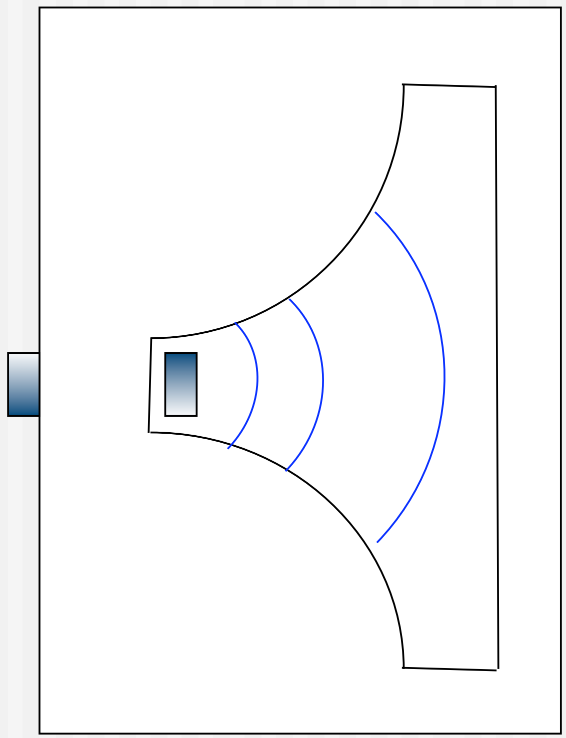
$B_x = B'y$

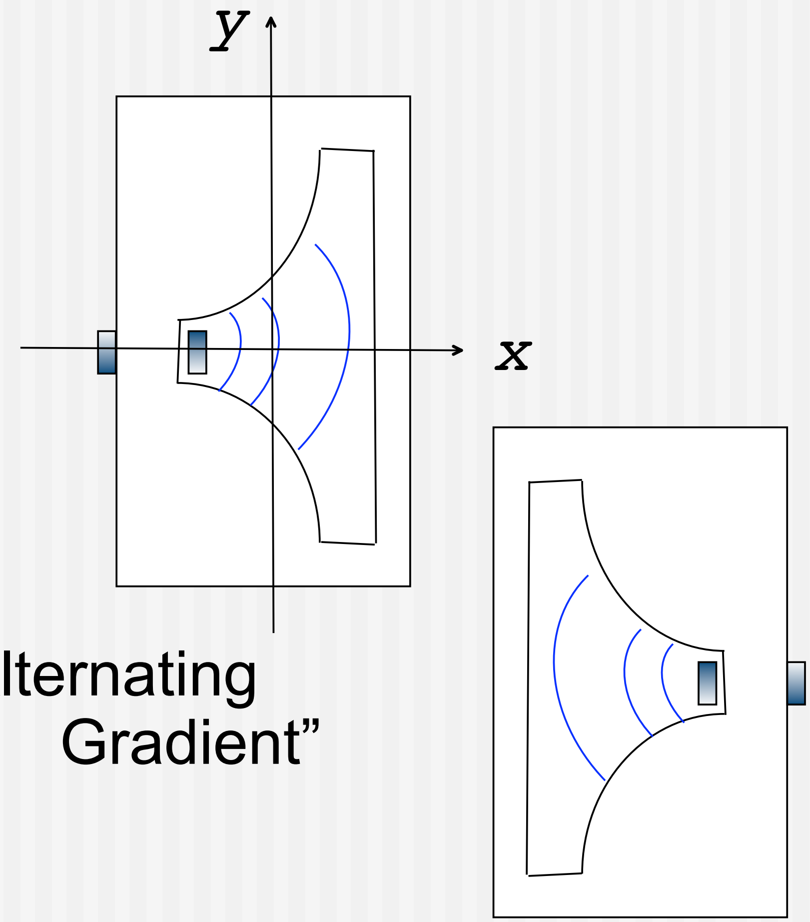
(Quadrupole Magnet)



Gradient Magnets

$$B_y = B_0 + B'x$$
$$B_x = B'y$$





“Alternating Gradient”

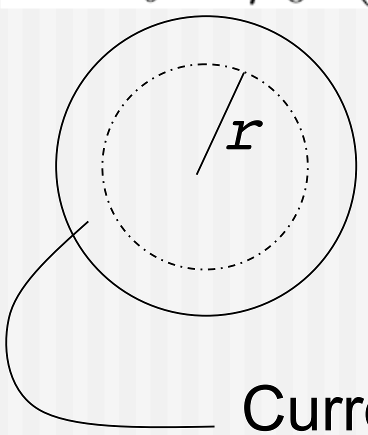
Iron-dominated Magnets

- Field Calculations of Simple Devices
  - Dipole magnet
  - Quadrupole magnet
- More complex designs

Superconducting Magnets

- Here, field is not shaped by iron pole tips, but rather is shaped by placement of the conductor
- Example: dipole magnet…

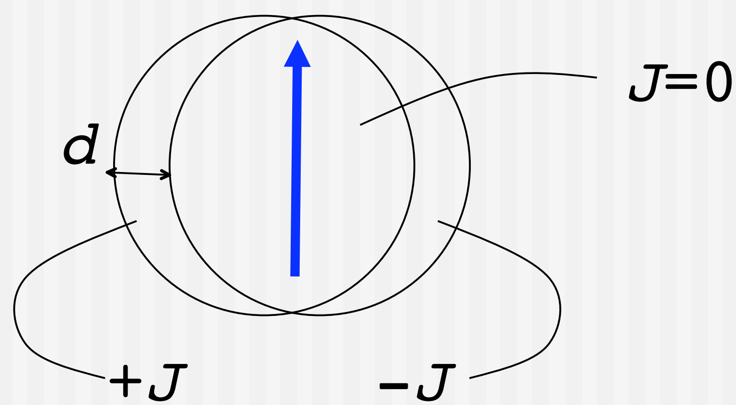
$$2\pi r B_\theta = \mu_0 J(\pi r^2)$$



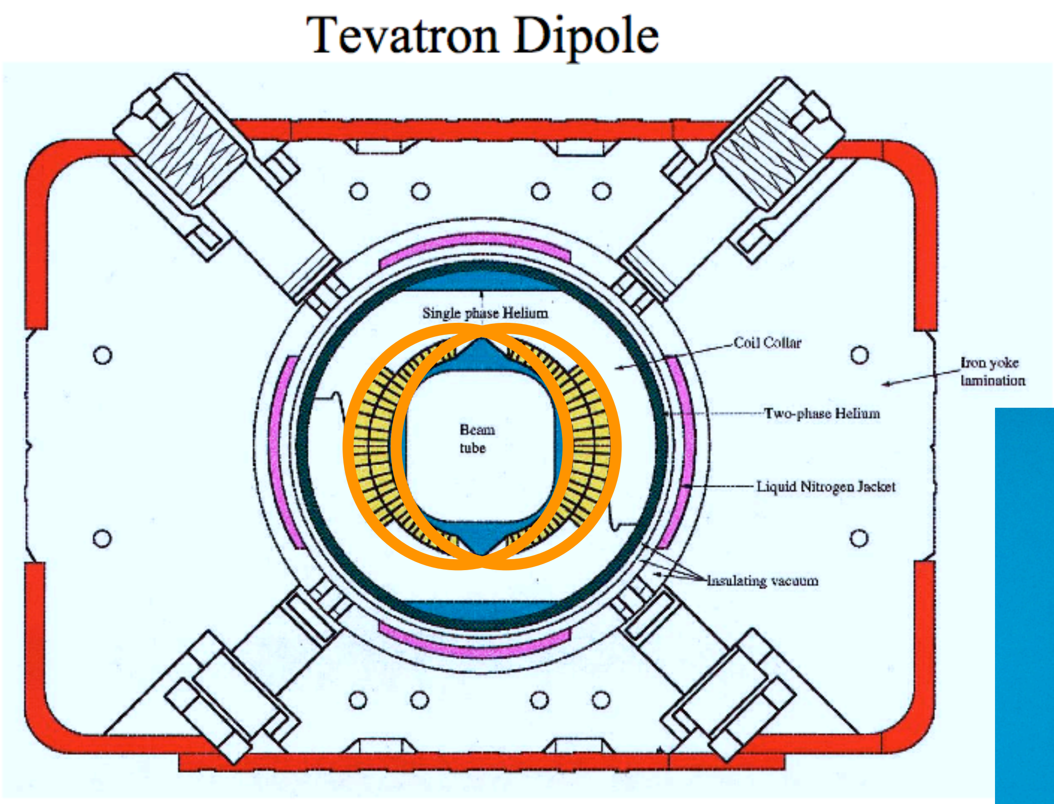
$$B_\theta = \frac{\mu_0 J}{2} r$$

Current density J

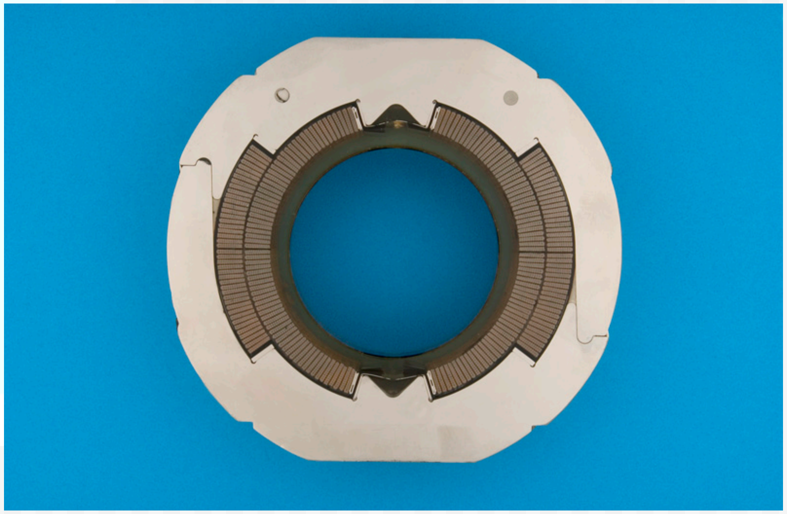
$$B_y = \frac{\mu_0 J}{2} d, \quad B_x = 0$$



Tevatron Dipole Magnet



“Cosine Theta” design



Superconducting Magnets

- Example:
  - J ~ 1000 A/mm^2, d ~ 1 cm

    ==> $B \sim (1/2)(4\pi\ 10^{-7})(1000*10^6)(10^{-2}) = 2\pi$ Tesla
- Tevatron -- ~4.4 Tesla
- SSC (above parameters) -- 6.6 Tesla
- LHC -- 8 Tesla
- LBNL model magnet -- 16 Tesla

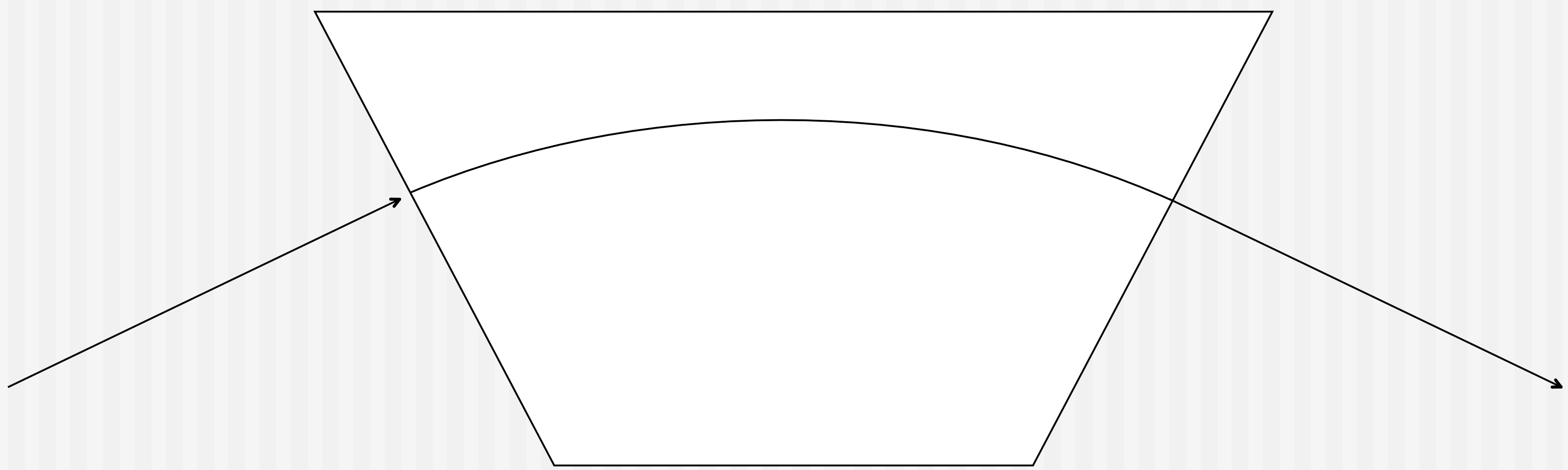
Note: Higher fields a “plus,” but field quality typically easier to control with iron pole tips shaping the field …

“Relaxation” Method

- When the field is independent of z, the average potential on circle parallel to x-y plane equals the potential at the center of the circle, provided no charge density in the region.
  - Generate “mesh” of points, with fixed potentials at the boundaries; generate “first guess” at values of potential at the mesh points
  - At each point, calculate average of potential of neighboring points; compare with the “guess”; alter the guess to split the difference.
  - Repeat, until “converge” to a solution for which the average of the neighbors equals each “guess” value (to a certain accuracy). From the resulting potential, compute the field.

Sector Magnets

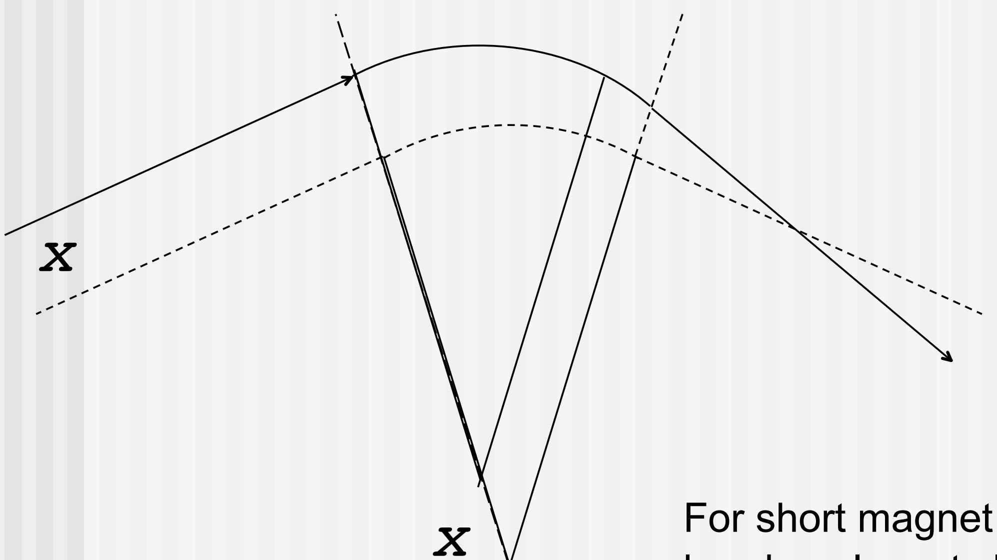
- Sector Dipole Magnet: "edge" of magnetic field is perpendicular to incoming/outgoing design trajectory:



Field points "*out of the page*"

Sector Magnets & Sector Focusing

- Incoming ray displaced from ideal trajectory will experience more/less bending field, thus is "focused" toward axis in the *bend plane*:



$$\text{Extra path length} = ds = d\theta\ x$$
$$\text{so extra bend angle} = dx \ \ = -ds/\rho$$
$$dx \ \ = -(d\theta/\rho)x = -(1/\rho^2)x\ ds$$
$$\text{or, } x \ \ = -(1/\rho^2)x$$

$$\text{Thus, } K_x = 1/\rho^2,\ K_y = 0.$$

(as seen previously, with B' = 0)

For short magnet with small bend angle, acts like lens in the bend plane with

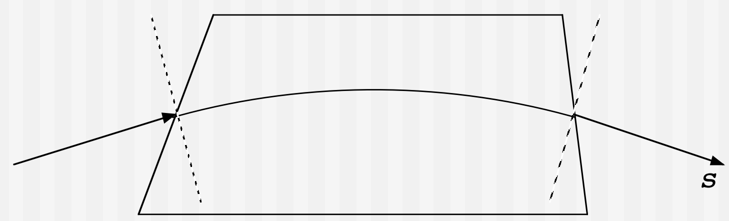
$$\frac{1}{f_x} = \frac{\theta}{\rho}$$

Edge Focusing

- In an ideal *sector magnet*, the magnetic field begins/ends exactly at $s = 0, L$ independent of transverse coordinates x,y relative to the design trajectory.
  - *i.e.*, the face of the magnet is perpendicular to the design trajectory at entrance/exit

Edge Focusing

- However, could (and often do) have the faces at angles *w.r.t.* the design trajectory -- provides “edge focusing”



- Since our transverse coordinate *x* is everywhere perpendicular to *s*, then a particle entering with an offset will see more/less bending at the interface...
- more later...

Some words on Space Charge

- For most of this course, will neglect the force on a particle due to the presence of surrounding particles
  - Fields within a uniform “bunch”
  - Fields within a Gaussian “bunch”
- will say more about this next week

Homework for Tuesday

- Problem Set 1 -- Nos. 3, 4, 5, 8, and 10